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Baffle Aperture Design Model for Electron Bombardment Thrusters

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A simple theoretical model that can be used as an aid in the design of the baffle aperture region of a hollow cathode-equipped electron bombardment thruster is developed. The electron diffusion process through the baffle aperture is modeled according to the Bohm diffusion theory. The model is shown to agree consistently with experimental data for a multipole thruster geometry over substantial changes in thruster operating conditions. However, on a radial field thruster geometry the model yields values that are consistently smaller than the experimental results by a factor of two. The design usefulness of the model is limited by this factor-of-two variation observed over different thruster geometries and by the accuracy to which the plasma parameters required as inputs to the model can be specified.

Nomenclature

A = area, m^2
B = magnetic flux density, T
D = diffusion coefficient, m^2/s
E = electric field strength, V/m
e = electronic charge, 1.6×10^{-19} C
I = current, A
i = unit vector
j = current density, A/m^2
k = Boltzmann's constant, 1.38×10^{-23} J/K
m = mass, kg
n = particle density, m^{-3}
P = electron pressure, N/m^2
r = radial position, m
T = temperature, K
V = plasma potential, V
v = fluid velocity, m/s
z = axial position, m
ν = collision frequency, Hz
ω = cyclotron frequency, Hz

Superscript
$(\bar{\ })$ = average

Subscripts
A = anode
a = baffle aperture
B = Bohm diffusion
b = extracted ion beam
C = classical diffusion
c = cathode region
D = discharge
E = cathode emission
e = electron
i = ion
m = main discharge region
P = production
s = screen grid
w = cathode potential surfaces
θ = azimuthal direction
$\perp$ = direction normal to magnetic field lines
$\parallel$ = direction parallel to magnetic field lines

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Introduction

THE study of ion thruster phenomena has been ongoing for many years. In spite of this there are still areas concerning thruster operation that are not well understood. The development of advanced thruster designs strongly depends on an increased understanding of the governing physical processes. This paper deals with the development of a model pertaining to the baffle aperture region of a hollow cathode-equipped ion thruster. As discussed by Wells,[1] it is in the baffle aperture region that the electrons emitted by the cathode are accelerated to energies sufficient to ionize the neutral propellant atoms. This means that the efficiency of an electron bombardment thruster depends heavily on the processes ongoing in this region. In the past the design and optimization of the cathode pole piece/baffle assembly has been done largely by trial and error.[2-5] To minimize the number of these types of time-consuming variations an investigation was conducted to develop a physical model relating the baffle aperture geometry to thruster operating parameters. The resultant model relates the magnetic flux density, pole piece geometry, and plasma properties of the baffle aperture region to the magnitude of the current through the aperture. The predictions of the model are compared to experimental results obtained on two different thruster geometries in widely varying operating conditions.

Baffle Aperture Model

Theoretical Development

The approach to the problem of developing a theoretical model which relates the current through the baffle aperture to the geometry, magnetic field, and plasma properties in this region was to treat the electron population as a fluid. Only those forces which act on fluid elements were considered; individual electron motions as well as ion motions were neglected. The fluid model is appropriate for the following reasons: 1) there are far too many particles involved to follow individual particle motions; and 2) the magnetic field can play the role of collisions in the sense that it limits the electron freestreaming and forces the electrons to exhibit collective behavior. For motions perpendicular to the magnetic field, the fluid theory is a good approximation.[6] The fluid equation of motion for the electrons, known as the momentum transfer equation, is written,[7]

$$m_e n\left[\frac{\partial \boldsymbol{v}}{\partial t}+(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right]=-ne(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})-\nabla P-\boldsymbol{v}nm_e\nu \quad (1)$$

This equation determines the average velocity of the system of particles under consideration and assumes the pressure to be locally isotropic. Assuming there is no net change in the average fluid velocity and for steady-state conditions Eq. (1) becomes,

$$ne(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})+\nabla P+\boldsymbol{v}nm_e\nu=0 \quad (2)$$

The current density $\boldsymbol{j}$ is introduced through the equation,

$$\boldsymbol{j}=-n\boldsymbol{v}e \quad (3)$$

and using the definition of plasma potential in terms of the electric field, $\nabla V=-\boldsymbol{E}$, Eq. (2) can be written as,

$$ne\left(\nabla V+\frac{\boldsymbol{j}\times\boldsymbol{B}}{ne}\right)-\nabla P+\frac{n\nu m_e}{e}\boldsymbol{j}=0 \quad (4)$$

At this point is is appropriate to specify the coordinate system to be used. The coordinate system is chosen so that the axes are parallel or perpendicular to the magnetic field lines. This is illustrated in the cathode pole piece shown in Fig. 1. In this coordinate system the current density can be written as,

$$\boldsymbol{j}=j_\perp \boldsymbol{i}_\perp+j_\theta \boldsymbol{i}_\theta+j_\parallel \boldsymbol{i}_\parallel \quad (5)$$

and the magnetic field as,

$$\boldsymbol{B}=B\boldsymbol{i}_\parallel \quad (6)$$

where $\boldsymbol{i}_\perp$, $\boldsymbol{i}_\theta$, and $\boldsymbol{i}_\parallel$ are unit vectors in the perpendicular, azimuthal, and parallel directions, respectively.

Now, because the electron mobility along magnetic field lines is much greater than the mobility across the field, it is reasonable to assume that there are no steady-state potential or density gradients parallel to the magnetic field. Further, it is assumed that there are no steady-state potential or density gradients in the azimuthal direction due to symmetry. With these assumptions the net current densities in the parallel, azimuthal, and normal directions relative to the magnetic field can be solved for from Eq. (3) to yield,

$$j_\parallel=0 \quad (7)$$

$$j_\theta=(\omega/\nu)j_\perp \quad (8)$$

and

$$j_\perp=-\left[\frac{\nu e}{m_e(\nu^2+\omega^2)}\right](ne\nabla_\perp V-\nabla_\perp P) \quad (9)$$

where $\omega=eB/2\pi m_e$ is the electron cyclotron frequency. Equation (7) indicates that there is no net current flow parallel to the magnet field and Eq. (8) says that no current can flow out of the aperture $j_\perp$ unless there is also a current in the azimuthal direction.[1]

Finally Eq. (9) can be written in terms of the classical diffusion coefficient defined by,[8]

$$D_C=\left[\frac{\nu}{\nu^2+\omega^2}\right]\frac{kT}{m_e} \quad (10)$$

to yield,

$$j_\perp=-D_C\left(\frac{e}{kT}\right)(ne\nabla_\perp V-\nabla_\perp P) \quad (11)$$

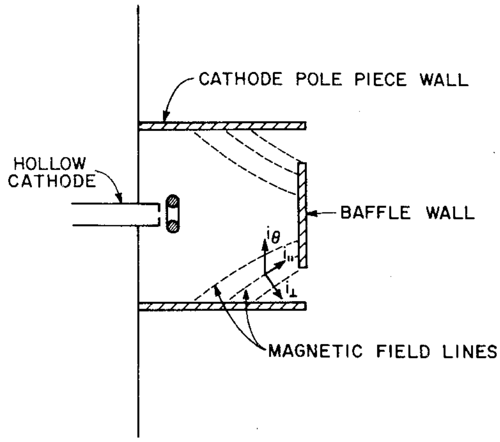

Fig. 1 Definition of local coordinate system.

For the magnetic field strengths and densities in the aperture region, it was found that,[9] $\omega/\nu \gg 1$, therefore ν^2 may be neglected relative to ω^2 in the denominator of Eq. (10) to obtain,

$$D_C=\frac{kTm_e\nu}{e^2B^2} \quad (12)$$

Diffusion coefficients calculated from Eq. (12) have often been found to be inadequate for use in typical ion thruster plasmas,[10,11] sometimes predicting coefficients orders of magnitude smaller than experimentally observed values. This failure of the classical theory is attributed to the enhancement of the diffusion mechanism by plasma turbulence.[12] A simple and well-known semiempirical approach to turbulent electron diffusion was given by Bohm[13] as,

$$D_B=kT/16eB \quad (13)$$

Although this relationship has been shown to work much better in an ion thruster than the classical theory,[10,11,14] it is still accurate only to within a factor of two or three.[13] In spite of this factor of two or three uncertainty, Bohm diffusion will be used in the remainder of the analysis because of its simplicity and relatively good agreement with experimental results. Using Eq. (13) in place of the classical diffusion coefficient in Eq. (11) yields,

$$j_\perp=-\frac{1}{16B}(ne\nabla_\perp V-\nabla_\perp P) \quad (14)$$

Now, making the change of notation, $\nabla_\perp \rightarrow \mathrm{d}/\mathrm{d}r$, which is appropriate when the net electron current flows radially out of a cylindrically symmetric baffle aperture,‡ Eq. (14) can be written,

$$j_\perp=-\frac{1}{16B}\left(ne\frac{\mathrm{d}V}{\mathrm{d}r}-\frac{\mathrm{d}P}{\mathrm{d}r}\right) \quad (15)$$

or

$$-16\int Bj_\perp\,\mathrm{d}r=e\int n\,\mathrm{d}V-\int \mathrm{d}P \quad (16)$$

where the integration is over the diffusion depth.[14]

The current density $j_\perp$ can be written as,

$$j_\perp=I/A \quad (17)$$

‡If the current flows in the axial direction then $\nabla_\perp \rightarrow \mathrm{d}/\mathrm{d}z$ should be used.

where A is the area through which the current I flows in the aperture. Neglecting ionizations in the aperture region the current I must be constant to satisfy continuity. The area through which the current flows, however, is not constant due to the cylindrical geometry and magnetic field configuration and, therefore, it must be left under the integral sign. Thus, Eq. (16) can be written as,

$$-16I\int B/A\,\mathrm{d}r = e\int n\,\mathrm{d}V - \int \mathrm{d}P \tag{18}$$

or

$$I = -\frac{e\int n\,\mathrm{d}V - \int \mathrm{d}P}{16\int B/A\,\mathrm{d}r} \tag{19}$$

This is the desired theoretical relationship between the current through the aperture and the geometry, magnetic field, and plasma properties around the baffle aperture region.

Baffle Aperture Current

In a normally operating thruster, electrons enter the discharge plasma in two ways. They are either emitted by the cathode or liberated in the ionization processes. In both cases the vast majority of electrons are constrained by the plasma boundary potential sheaths to leave the plasma only at the anode. That is, the plasma sheaths at all cathode potential surfaces are of sufficient magnitude to reflect all but the most energetic electrons in the tail of the Maxwellian distribution.[5] At the anode, however, the potential sheaths are such that the electrons are easily collected. Thus, both the electrons emitted by the cathode and those left behind by the ions which have been extracted into the beam are collected by the anode and contribute to the discharge current. In addition, there are also electrons left behind in the plasma by the ions which strike cathode potential surfaces. These electrons are also collected by the anode and subsequently go through the discharge power supply which sends them to the thruster's cathode potential surfaces where they neutralize the incoming ions. Therefore, the discharge current is seen to be the sum of electron currents from three different sources: electrons emitted by the cathode, those left behind by beam ions, and those left behind by ions striking cathode potential surfaces, so that,

$$I_D = I_E + I_b + I_w \tag{20}$$

Electrons left behind by ions which strike anode potential surfaces do not contribute to the discharge current since these electron-ion pairs are reunited at the anode surface directly without requiring the electrons to go through the discharge power supply. In Eq. (20), I_E is understood to be the net cathode emission current [comprised of electrons] emitted by the cathode and not subsequently collected by the keeper electrode. This current then (neglecting ionizations in the cathode discharge region) is approximately the electron current which flows through the baffle aperture and we can write,

$$I = I_D - I_b - I_w \tag{21}$$

Simplifications

Although Eq. (19) provides a theoretical relationship between the parameters of interest, it is undesirable in its present form because evaluation of the integrals on the right-hand side of this equation requires a detailed knowledge of the plasma properties through the aperture. However, since such information is not generally available before a thruster is built and tested, some simplification of Eq. (19) is in order. The second integral in the numerator of Eq. (19) can be integrated directly to yield,

$$\int \mathrm{d}P = \Delta P \tag{22}$$

where ΔP is simply the difference in electron pressure between the main and cathode discharge regions. Simplification of the term $\int n\,\mathrm{d}V$ in Eq. (19) is a little less straightforward. Figure 2 shows the density and potential variation through a typical baffle aperture. (These measurements were actually made in the radial field thruster to be described later. In this thruster the direction perpendicular to the magnetic field (horizontal axis) is the radial direction.) These and other data not presented suggest that a reasonable approximation to the term can be obtained from,

$$\int n\,\mathrm{d}V = \bar{n}\Delta V \tag{23}$$

where ΔV is the change in potential across the aperture and $\bar{n}$ the average value of the density defined by,

$$\bar{n} = (n_m + n_c)/2 \tag{24}$$

Combining Eqs. (19), (22), and (23) yields,

$$I = -\frac{e\bar{n}\Delta V - \Delta P}{16\int B/A\,\mathrm{d}r} \tag{25}$$

Equation (25) is an approximate version of Eq. (19) and requires plasma data from only two points, the cathode and main discharge regions, as inputs. The integral in the denominator of Eq. (25) was left intact because the strength and configuration of the magnetic field, as well as the aperture area, are under the direct control of the designer and thus the integral can easily be specified, a priori.

Apparatus

Although the aperture region was of prime consideration for this study, it must be investigated as part of a complete thruster. Therefore, the two different thruster configurations described below were tested.

Tests were conducted on the 14 cm diam radial field thruster shown schematically in Fig. 3. For this thruster and cathode pole piece geometry, the electron current emitted by the cathode flows axially into the cathode discharge chamber. From there the current passes radially out of the cathode chamber across the aperture region magnetic field. The electrons diffuse across the magnetic field lines in the baffle aperture region until they reach the field line which carries them into the main discharge region. This field line is called

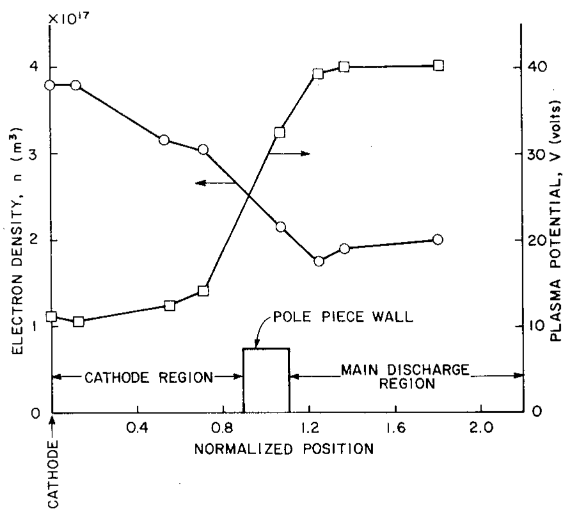

Fig. 2 Typical plasma density and potential variations through the baffle aperture.

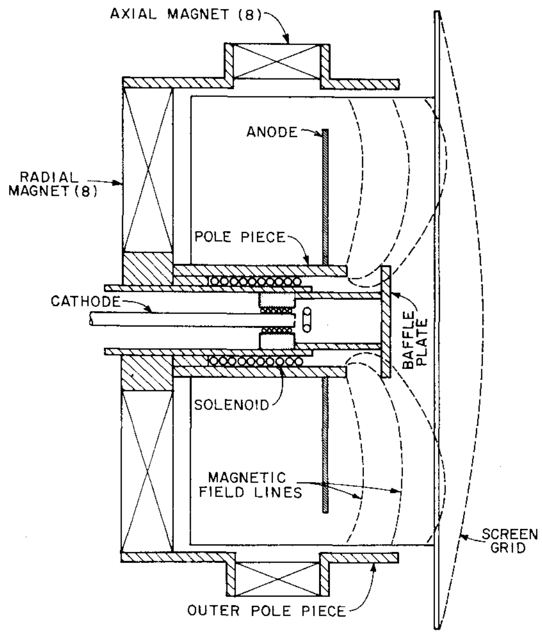

Fig. 3 Radial field thruster equipped with magnetic baffle.

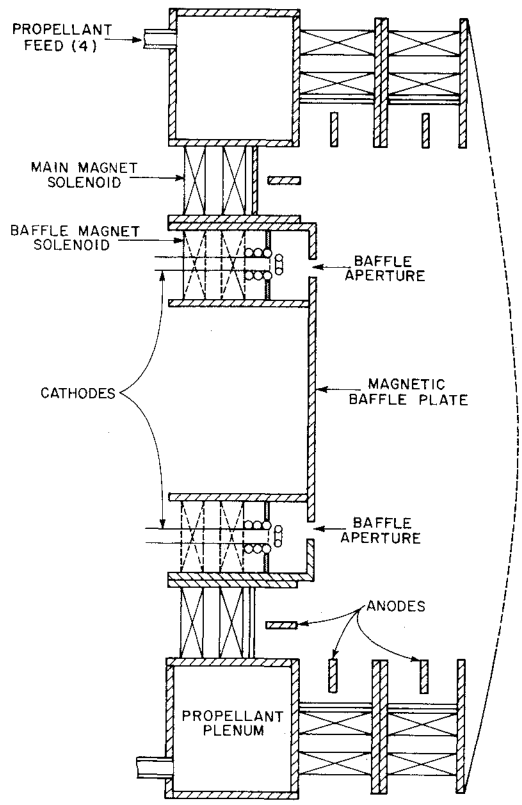

Fig. 4 Multipole thruster with large cathode region/magnetic baffle assembly.

the critical magnetic field line. Eight radially oriented electromagnets were used to control the strength of the magnetic field in the main discharge region and a 12 turn magnetic baffle solenoid was used to vary the magnetic field strength in the aperture. Two cylindrical Langmuir probes were used to measure the plasma properties on each side of the aperture. Both probes were constructed of tantalum wire and were supported from quartz tube insulators. The cathode region probe was positioned at a radial distance approximately equal to the keeper radius at the axial midpoint of the aperture gap. The main region probe was normally positioned just outside of the critical magnetic field line. However, it could also be swept radially through the aperture to facilitate the measurement of plasma properties as a function of radial position.

Additional tests were conducted on the 15 cm diam multipole thruster shown schematically in Fig. 4. This thruster was outfitted with an abnormally large cathode region/magnetic baffle assembly which provided a radically different geometry from that of the radial field thruster on which to test the model. The direction of electron current flow through the aperture is also indicated in Fig. 4. Again, two probes, similar in construction to the probes used on the radial thruster, were used to measure the plasma properties on each side of the aperture. Because the magnetic field created by the baffle electromagnets was, for this thruster geometry, concentrated in the aperture region only, the cathode region probe was positioned at the radius of the aperture rather than over the keeper as it was for the radial field thruster. The positions of both probes are indicated in Fig. 4. In addition, the cathode was electrically isolated from the thruster body so that the net electron current emitted by the cathode could be measured directly during operation.

Procedure

The following experimental procedure was implemented for the collection of data on the radial field thruster. Before each test run, the baffle aperture area was set by adjusting the distance between the downstream edge of the cathode pole piece and the baffle plate. Three different aperture areas were tested. During thruster operation the current through the aperture was indirectly varied by changing the current through the discharge power supply. In addition, the magnetic field strengths in the aperture and main discharge regions were varied by changing the currents through the radial and baffle electromagnets. The magnetic flux density in the main discharge region was varied at $7\text{-}45 \times 10^{-4}$ T, while the field in the aperture region was varied over the range of $9\text{-}110 \times 10^{-4}$ T. Finally, the cathode and main neutral flow rates were varied over the ranges of 80-200 and 400-600 mA, respectively. These changes resulted in an order-of-magnitude variation in the beam current of 40-485 mA. At each discharge current, magnetic field, and aperture area setting tested the following data were collected. Langmuir probe§ traces, from which the plasma properties of interest can be determined, were taken in both the cathode and main discharge regions and the discharge current, discharge voltage, beam current, keeper voltage, propellant flow rate, and electromagnet currents were recorded. After shutdown of the thruster and its subsequent removal from the vacuum system, the magnetic flux density through the aperture was measured with a two-axis Gauss meter. The magnetic field strength was measured for each setting of radial and baffle electromagnet current at which Langmuir probe traces were recorded.

The procedure used on the radial field thruster was also followed for the collection of data on the multipole thruster, with the exception that the net cathode emission current was also recorded at each operating condition tested. A summary of the parameters varied in the tests on both thrusters is provided in Table 1.

Results and Discussion

To test the validity of Eq. (25) experimentally, it was found more convenient for the presentation of the results to rewrite it in the form,

$$\int \frac{B}{A} \mathrm{d}r = -\frac{l}{16I} [e\bar{n}\Delta V - \Delta P] \tag{26}$$

§Both Langmuir probes were cleaned by ion bombardment[15] before any traces were taken. Probe traces were analyzed using a program developed by Beattie.[16]

Table 1 Test conditions

Test No.	Thruster	Aperture area, 10^{-4} m^2	Flow rate range, mA eq. Cathode	Main	Aperture current, A	Aperture magnetic flux density, 10^{-4} T
1	Radial	5.7	153-200	650	3.0-4.5	9-111
2	Radial	6.9	102-158	475-510	2.3-3.4	15-107
3	Radial	9.1	80-100	406-500	2.2-3.5	13-100
4	Multipole	11.0	200-250	400-450	1.5-3.2	10-33

Thus, the value of the integral, $\int B/A\,dr$, computed from Eq. (26) using measured plasma data can be compared to the value determined from measuring the magnetic field and area variation through the aperture directly. The detailed procedure used to compute this integral from the magnetic flux density and area measurements is described in Ref. 9.

The results of the comparison of the integrals calculated by Eq. (26) to the directly measured values for the radial field thruster is given in Fig. 5, where the current through the baffle aperture was taken to be equal to the net cathode emission current which was calculated according to Eq. (21). The solid line represents a linear least squares curve fit forced to go through the origin. This line has a correlation coefficient of 0.83 and a slope of ½. The different symbols represent the first three test configurations listed in Table 1. The model is seen to yield remarkably consistent results, even over extreme changes in thruster operating conditions. Ideally, however, the curve fit should have a slope of unity if the model is exactly correct. As it is, the model predicts values of the integral that are a factor of two low. It is noted, however, that this is within the accuracy of the Bohm diffusion coefficient. Consequently, a straightforward method to obtain good agreement between the model and experiment would be to change the constant 1/16 in the Bohm diffusion formula. The value 1/16 has no theoretical justification but is an empirical number agreeing with most experiments to within a factor of two or three.[17] An experimental fit of this constant by Spitzer[18] yielded a value of 0.21 and Yoshikawa and Rose[12] give the constant in the range 0.031 and 0.063, depending on the magnitude of the density fluctuations. However, both of these studies as well as Bohm's original work were conducted at magnetic field strengths and plasma densities much greater than those normally found in an ion thruster. In the few diffusion studies actually conducted in operating ion thrusters[10,11,14] which employed Bohm diffusion, only Robinson[19] obtained results which were good to better than a factor of two or three. Consequently, it is not too surprising that the coefficient is off by a factor of two. A more important result is the minimum of scatter observed over substantial changes in operating parameters. Good agreement between the model and experiment would be obtained in this case for an empirical constant of ⅛ rather than 1/16.

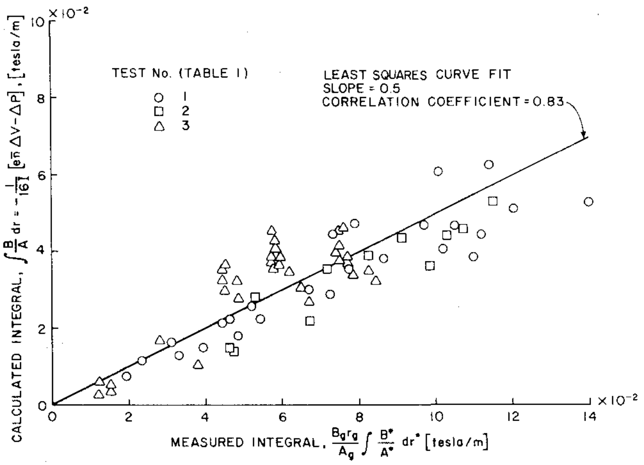

Fig. 5 Comparison of calculated integrals with the measured values for the radial field thruster.

It was of interest to test the model on a radically different cathode region and thruster geometry from that of the radial field thruster. The multipole thruster of Fig. 4 meets this requirement. The major differences between this thruster geometry and that of the radial field thruster are the following. First, the current flows essentially in the axial direction through the aperture rather than radially. Second, the large size of the cathode region means that the mean diameter of the aperture is considerably larger in this case as is the distance between the cathode itself and the aperture. This makes the coupling between the cathode and the anode more difficult. Finally, the main discharge region is essentially field free, making less obvious the location of the critical magnetic field line from which electrons find access to the main discharge. Due to the axial current flow direction through the aperture for this thruster, Eq. (25) must be written in the form,

$$\int \frac{B}{A}\,dz = -\frac{1}{16I}\left(e\bar{n}\Delta V - \Delta P\right) \tag{27}$$

where z is in the axial direction. A comparison of results obtained from the calculations of Eq. (27) and the measured integrals is given in Fig. 6. For this case it is seen that the theory predicts values that are scattered about the line of perfect correlation with no change of the empirical constant 1/16 required. In addition, the solid symbols in Fig. 6 indicate that the theory is independent of whether or not the high voltage is on as would be expected. However, it is not clear why the model should agree correctly for this thruster

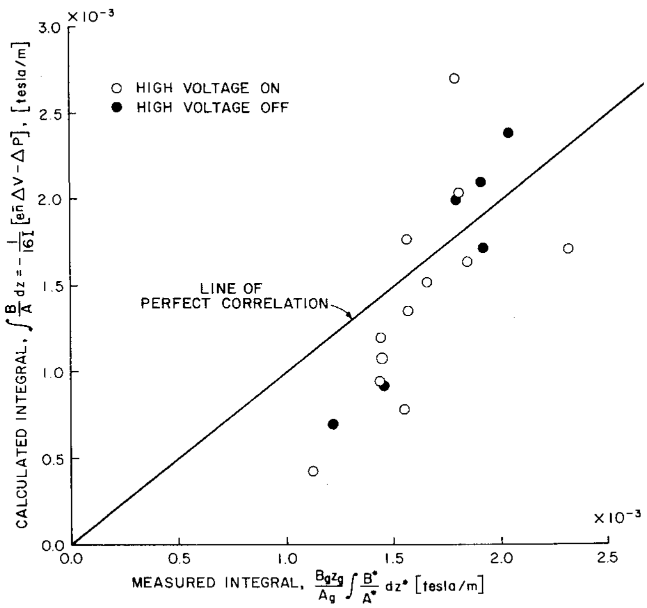

Fig. 6 Comparison of calculated integrals with the measured values for the multipole thruster.

geometry and be off by a factor of two for the radial field thruster geometry since one would expect the diffusion processes to be similar in both cases. It is possible, however, that a change such as that in the cathode pole piece geometry could change the nature of the turbulence which affects this empirical constant.

Inspection of Eq. (25) indicates that the electron current is driven through the baffle aperture under the influence of two forces; the force due to the potential difference $e\bar{n}\Delta V$ and the force due to the pressure difference ΔP. Under all observed thruster operating conditions, the force due to the potential difference was seen to be by far the dominant force driving the electron diffusion in this region.

Conclusions

An ion and electron current balance indicates that the current through the baffle aperture is approximately equal to the discharge current minus the sum of the beam current and the ion current to cathode potential surfaces. The Bohm diffusion theory may be used to model the diffusion of electrons through this region. The classical diffusion theory typically predicts electron diffusion coefficients in the aperture which are at least an order of magnitude too small. The simplified theoretical model of the baffle aperture region agrees correctly with the experimental results obtained on the multipole thruster geometry. On the radial field thruster the model differs consistently from the experimental results by a factor of two. Finally, the potential difference across the baffle aperture is the dominant force driving the electron diffusion across the magnetic field lines in this region.

Acknowledgment

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References

[1] Wells, A.A., "Current Flow Across a Plasma 'Double Layer' in a Hollow Cathode Ion Thruster," AIAA Paper 72-418, 1972.

[2] Poeschel, R.L., Ward, J.W., and Knauer, W., "Study and Optimization of 15-cm Kaufman Thruster Discharges," AIAA Paper 69-257, 1969.

[3] King, H.J., Poeschel, R.L., and Ward, J.W., "A 30-cm Low-Specific Impulse, Hollow-Cathode, Mercury Thruster," AIAA Paper 69-300, 1969.

[4] Palumbo, G. et al., "Effect of Geometry and Field Inside the Pole Piece in an Electron Bombardment Thruster," AIAA Paper 70-1088, 1970.

[5] Longhurst, G.R. and Wilbur, P.J., "Plasma Property and Performance Prediction for Mercury Ion Thrusters," AIAA Paper 79-2054, 1979.

[6] Chen, F.F., *Introduction to Plasma Physics,* Plenum Press, New York, 1974, p. 57.

[7] Unman, M.A., *Introduction to Plasma Physics,* McGraw-Hill Book Co., New York, 1964, p. 168.

[8] Jahn, R.G., *Physics of Electric Propulsion,* McGraw-Hill Book Co., New York, 1968, Chap. 5.

[9] Brophy, J.R., "Baffle Aperture Design Study of Hollow Cathode Equipped Ion Thrusters," M.S. Thesis, Dept. of Mechanical Engineering, Colorado State University, Ft. Collins, Oct. 1980.

[10] Isaacson, G.C., "Multipole Gas Thruster Design," Ph.D. Thesis, Dept. of Physics, Colorado State University, Fort Collins, June 1977.

[11] Longhurst, G.R., "The Diffusion of Electrons in Ion Thrusters," *Mercury Ion Thruster Research—1977,* edited by Paul Wilbur, NASA CR-135317, Dec. 1977.

[12] Yoshikawa, S. and Rose, D.J., "Anomalous Diffusion of a Plasma Across a Magnetic Field," *Physics of Fluids,* Vol. 5, March 1962, p. 334-341.

[13] Bohm, D., "The Use of Probes for Plasma Exploration in Strong Magnetic Fields," *Characteristics of Electrical Discharges in Magnetic Fields,* edited by A. Guthrie and R.K. Wakerling, McGraw-Hill Book Co., New York, 1949.

[14] Kaufman, H.R. and Robinson, R.S., "Plasma Processes in Inert Gas Thrusters," AIAA Paper 79-2055, 1979.

[15] Wehner, G. and Medicus, L., "Reliability of Probe Measurements in Hot Cathode Gas Diodes," *Journal of Applied Physics,* Vol., 23, Sept. 1952, pp. 1035-1046.

[16] Beattie, J.R., "Numerical Procedure for Analyzing Langmuir Probe Data," *AIAA Journal,* Vol. 13, July 1975, pp. 950-952.

[17] Chen, F.F., *Introduction to Plasma Physics,* Plenum Press, New York, 1974, Chap. 5, p. 169.

[18] Spitzer, L., "Particle Diffusion Across a Magnetic Field," *Physics of Fully Ionized Gases,* Interscience Publishers, Inc., New York, 1956.

[19] Robinson, R.S., "Physical Processes in Ion Beam Sputtering," Ph.D. Thesis, Dept. of Physics, Colorado State University, Fort Collins, March 1979.